

Lecture M100904

Integration By Partial Fraction

Zarif Ahmad Razin Bhuiyan

5th semester, Mechanical Engineering
Bangladesh University of Engineering And Technology (BUET)

January 13, 2022

Problem: 1(vii)

$$\int \frac{2x + 3}{x^3 - x} dx$$

Problem: 1(vii)

$$\int \frac{2x + 3}{x^3 - x} dx$$

Solution:

Step1: Partial Fraction

Problem: 1(vii)

$$\int \frac{2x+3}{x^3-x} dx$$

Solution:

Step1: Partial Fraction

$$\begin{aligned}\frac{2x+3}{x^3-x} &= \frac{2x+3}{x(x^2-1)} \\ &= \frac{2x+3}{x(x+1)(x-1)} \\ \therefore \frac{2x+3}{x^3-x} &\equiv \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1} \dots (1)\end{aligned}$$

Problem: 1(vii)

$$\int \frac{2x+3}{x^3-x} dx$$

Solution:

Step1: Partial Fraction

$$\begin{aligned} \frac{2x+3}{x^3-x} &= \frac{2x+3}{x(x^2-1)} \\ &= \frac{2x+3}{x(x+1)(x-1)} \\ \therefore \frac{2x+3}{x^3-x} &\equiv \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1} \dots (1) \end{aligned}$$

Multiplying both side by x and taking limit $x \rightarrow 0$

$$\begin{aligned} \lim_{x \rightarrow 0} \left[\frac{2x+3}{x^3-x} \times x \right] &= \lim_{x \rightarrow 0} \left[\frac{A}{x} \times x \right] + \lim_{x \rightarrow 0} \left[\frac{Bx}{x+1} \right] \\ &\quad + \lim_{x \rightarrow 0} \left[\frac{Cx}{x-1} \right] \end{aligned}$$

Problem: 1(vii)

$$\int \frac{2x+3}{x^3-x} dx$$

Solution:

Step1: Partial Fraction

$$\begin{aligned} \frac{2x+3}{x^3-x} &= \frac{2x+3}{x(x^2-1)} \\ &= \frac{2x+3}{x(x+1)(x-1)} \\ \therefore \frac{2x+3}{x^3-x} &\equiv \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1} \dots (1) \end{aligned}$$

Multiplying both side by x and taking limit $x \rightarrow 0$

$$\begin{aligned} \lim_{x \rightarrow 0} \left[\frac{2x+3}{x^3-x} \times x \right] &= \lim_{x \rightarrow 0} \left[\frac{A}{x} \times x \right] + \lim_{x \rightarrow 0} \left[\frac{Bx}{x+1} \right] \\ &\quad + \lim_{x \rightarrow 0} \left[\frac{Cx}{x-1} \right] \end{aligned}$$

$$\text{or, } \lim_{x \rightarrow 0} \left[\frac{2x+3}{x(x+1)(x-1)} \times x \right] = \lim_{x \rightarrow 0} A + 0 + 0$$

$$\text{or, } \lim_{x \rightarrow 0} \left[\frac{2x+3}{(x+1)(x-1)} \right] = A$$

$$\text{or, } A = \lim_{x \rightarrow 0} \left[\frac{2x+3}{(x+1)(x-1)} \right]$$

$$\text{or, } A = \frac{2 \times 0 + 3}{(0+1)(0-1)}$$

$$\therefore A = -3$$

Similarly, multiplying both side of (1) by $(x+1)$ and taking limit $x \rightarrow -1$

$$B = \frac{1}{2}$$

and, multiplying both side of (1) by $(x-1)$ and taking limit $x \rightarrow 1$ will be found $C = \frac{5}{2}$

Exercise 10.4

$$\frac{2x+3}{x(x+1)(x-1)} = \frac{-3}{x} + \frac{1}{2} \frac{1}{x+1} + \frac{5}{2} \frac{1}{x-1}$$

Exercise 10.4

$$\frac{2x+3}{x(x+1)(x-1)} = \frac{-3}{x} + \frac{1}{2} \frac{1}{x+1} + \frac{5}{2} \frac{1}{x-1}$$

Step 1: Partial Fraction (Short Method)

$$\begin{aligned} \frac{2x+3}{x(x+1)(x-1)} &= \frac{2 \times 0 + 3}{x(0+1)(0-1)} \\ &+ \frac{2 \times (-1) + 3}{(-1)(x+1)(-1-1)} \\ &+ \frac{2 \times 1 + 3}{1(1+1)(x-1)} \end{aligned}$$

Exercise 10.4

$$\frac{2x+3}{x(x+1)(x-1)} = \frac{-3}{x} + \frac{1}{2} \frac{1}{x+1} + \frac{5}{2} \frac{1}{x-1}$$

Step 1: Partial Fraction (Short Method)

$$\begin{aligned} \frac{2x+3}{x(x+1)(x-1)} &= \frac{2 \times 0 + 3}{x(0+1)(0-1)} \\ &\quad + \frac{2 \times (-1) + 3}{(-1)(x+1)(-1-1)} \\ &\quad + \frac{2 \times 1 + 3}{1(1+1)(x-1)} \\ &= \frac{-3}{x} + \frac{1}{2} \frac{1}{x+1} + \frac{5}{2} \frac{1}{x-1} \end{aligned}$$

Exercise 10.4

Step 2: Integration

$$I = \int \frac{2x + 3}{x(x + 1)(x - 1)} dx$$

Step 1: Partial Fraction (Short Method)

$$\begin{aligned} \frac{2x + 3}{x(x + 1)(x - 1)} &= \frac{2 \times 0 + 3}{x(0 + 1)(0 - 1)} \\ &\quad + \frac{2 \times (-1) + 3}{(-1)(x + 1)(-1 - 1)} \\ &\quad + \frac{2 \times 1 + 3}{1(1 + 1)(x - 1)} \\ &= \frac{-3}{x} + \frac{1}{2} \frac{1}{x + 1} + \frac{5}{2} \frac{1}{x - 1} \end{aligned}$$

Exercise 10.4

$$\frac{2x+3}{x(x+1)(x-1)} = \frac{-3}{x} + \frac{1}{2} \frac{1}{x+1} + \frac{5}{2} \frac{1}{x-1}$$

Step 1: Partial Fraction (Short Method)

$$\begin{aligned}\frac{2x+3}{x(x+1)(x-1)} &= \frac{2 \times 0 + 3}{x(0+1)(0-1)} \\ &\quad + \frac{2 \times (-1) + 3}{(-1)(x+1)(-1-1)} \\ &\quad + \frac{2 \times 1 + 3}{1(1+1)(x-1)} \\ &= \frac{-3}{x} + \frac{1}{2} \frac{1}{x+1} + \frac{5}{2} \frac{1}{x-1}\end{aligned}$$

Step 2: Integration

$$\begin{aligned}I &= \int \frac{2x+3}{x(x+1)(x-1)} dx \\ &= \int \left(\frac{-3}{x} + \frac{1}{2} \times \frac{1}{x+1} + \frac{5}{2} \times \frac{1}{x-1} \right) dx\end{aligned}$$

Exercise 10.4

$$\frac{2x+3}{x(x+1)(x-1)} = \frac{-3}{x} + \frac{1}{2} \frac{1}{x+1} + \frac{5}{2} \frac{1}{x-1}$$

Step 1: Partial Fraction (Short Method)

$$\begin{aligned}\frac{2x+3}{x(x+1)(x-1)} &= \frac{2 \times 0 + 3}{x(0+1)(0-1)} \\ &\quad + \frac{2 \times (-1) + 3}{(-1)(x+1)(-1-1)} \\ &\quad + \frac{2 \times 1 + 3}{1(1+1)(x-1)} \\ &= \frac{-3}{x} + \frac{1}{2} \frac{1}{x+1} + \frac{5}{2} \frac{1}{x-1}\end{aligned}$$

Step 2: Integration

$$\begin{aligned}I &= \int \frac{2x+3}{x(x+1)(x-1)} dx \\ &= \int \left(\frac{-3}{x} + \frac{1}{2} \times \frac{1}{x+1} + \frac{5}{2} \times \frac{1}{x-1} \right) dx \\ &= \int \frac{-3}{x} dx + \int \frac{\frac{1}{2}}{x+1} dx + \int \frac{\frac{5}{2}}{x-1} dx\end{aligned}$$

$$\frac{2x+3}{x(x+1)(x-1)} = \frac{-3}{x} + \frac{1}{2} \frac{1}{x+1} + \frac{5}{2} \frac{1}{x-1}$$

Step 1: Partial Fraction (Short Method)

$$\begin{aligned} \frac{2x+3}{x(x+1)(x-1)} &= \frac{2 \times 0 + 3}{x(0+1)(0-1)} \\ &\quad + \frac{2 \times (-1) + 3}{(-1)(x+1)(-1-1)} \\ &\quad + \frac{2 \times 1 + 3}{1(1+1)(x-1)} \\ &= \frac{-3}{x} + \frac{1}{2} \frac{1}{x+1} + \frac{5}{2} \frac{1}{x-1} \end{aligned}$$

Step 2: Integration

$$\begin{aligned} I &= \int \frac{2x+3}{x(x+1)(x-1)} dx \\ &= \int \left(\frac{-3}{x} + \frac{1}{2} \times \frac{1}{x+1} + \frac{5}{2} \times \frac{1}{x-1} \right) dx \\ &= \int \frac{-3}{x} dx + \int \frac{\frac{1}{2}}{x+1} dx + \int \frac{\frac{5}{2}}{x-1} dx \\ &= -3 \int \frac{1}{x} dx + \frac{1}{2} \int \frac{1}{x+1} + \frac{5}{2} \int \frac{1}{x-1} dx \end{aligned}$$

$$\frac{2x+3}{x(x+1)(x-1)} = \frac{-3}{x} + \frac{1}{2} \frac{1}{x+1} + \frac{5}{2} \frac{1}{x-1}$$

Step 1: Partial Fraction (Short Method)

$$\begin{aligned} \frac{2x+3}{x(x+1)(x-1)} &= \frac{2 \times 0 + 3}{x(0+1)(0-1)} \\ &\quad + \frac{2 \times (-1) + 3}{(-1)(x+1)(-1-1)} \\ &\quad + \frac{2 \times 1 + 3}{1(1+1)(x-1)} \\ &= \frac{-3}{x} + \frac{1}{2} \frac{1}{x+1} + \frac{5}{2} \frac{1}{x-1} \end{aligned}$$

Step 2: Integration

$$\begin{aligned} I &= \int \frac{2x+3}{x(x+1)(x-1)} dx \\ &= \int \left(\frac{-3}{x} + \frac{1}{2} \times \frac{1}{x+1} + \frac{5}{2} \times \frac{1}{x-1} \right) dx \\ &= \int \frac{-3}{x} dx + \int \frac{\frac{1}{2}}{x+1} dx + \int \frac{\frac{5}{2}}{x-1} dx \\ &= -3 \int \frac{1}{x} dx + \frac{1}{2} \int \frac{1}{x+1} + \frac{5}{2} \int \frac{1}{x-1} dx \\ &= -3 \ln x + \frac{1}{2} \ln(x+1) + \frac{5}{2} \ln(x-1) + c \end{aligned}$$

Problem 2(v):

$$\int \frac{2x + 3}{x^3 + x^2 - 2x} dx$$

Solution:

Step 1: Partial Fraction

$$\frac{2x + 3}{x^3 + x^2 - 2x} = \frac{2x + 3}{x(x^2 + x - 2)}$$

Problem 2(v):

$$\int \frac{2x + 3}{x^3 + x^2 - 2x} dx$$

Solution:

Step 1: Partial Fraction

$$\begin{aligned} \frac{2x + 3}{x^3 + x^2 - 2x} &= \frac{2x + 3}{x(x^2 + x - 2)} \\ &= \frac{2x + 3}{x(x^2 + 2x - x - 2)} \end{aligned}$$

Problem 2(v):

$$\int \frac{2x + 3}{x^3 + x^2 - 2x} dx$$

Solution:

Step 1: Partial Fraction

$$\begin{aligned}\frac{2x + 3}{x^3 + x^2 - 2x} &= \frac{2x + 3}{x(x^2 + x - 2)} \\ &= \frac{2x + 3}{x(x^2 + 2x - x - 2)} \\ &= \frac{2x + 3}{x(x(x + 2) - 1(x + 2))}\end{aligned}$$

Problem 2(v):

$$\int \frac{2x + 3}{x^3 + x^2 - 2x} dx$$

Solution:

Step 1: Partial Fraction

$$\begin{aligned}\frac{2x + 3}{x^3 + x^2 - 2x} &= \frac{2x + 3}{x(x^2 + x - 2)} \\ &= \frac{2x + 3}{x(x^2 + 2x - x - 2)} \\ &= \frac{2x + 3}{x(x(x + 2) - 1(x + 2))} \\ &= \frac{2x + 3}{x(x + 2)(x - 1)}\end{aligned}$$

Problem 2(v):

$$\int \frac{2x + 3}{x^3 + x^2 - 2x} dx$$

Solution:

Step 1: Partial Fraction

$$\begin{aligned} \frac{2x + 3}{x^3 + x^2 - 2x} &= \frac{2x + 3}{x(x^2 + x - 2)} \\ &= \frac{2x + 3}{x(x^2 + 2x - x - 2)} \\ &= \frac{2x + 3}{x(x(x + 2) - 1(x + 2))} \\ &= \frac{2x + 3}{x(x + 2)(x - 1)} \\ &\equiv \frac{2 \times 0 + 3}{x(0 + 2)(0 - 1)} \\ &\quad + \frac{2 \times (-2) + 3}{(-2)(x + 2)(-2 - 1)} + \frac{2 \times 1 + 3}{1(1 + 2)(x - 1)} \end{aligned}$$

Problem 2(v):

$$\int \frac{2x+3}{x^3+x^2-2x} dx$$

$$\frac{2x+3}{x^3+x^2-2x} = \frac{-\frac{3}{2}}{x} + \frac{\frac{1}{6}}{x+2} + \frac{\frac{5}{3}}{x-1}$$

Solution:

Step 1: Partial Fraction

$$\begin{aligned} \frac{2x+3}{x^3+x^2-2x} &= \frac{2x+3}{x(x^2+x-2)} \\ &= \frac{2x+3}{x(x^2+2x-x-2)} \\ &= \frac{2x+3}{x(x(x+2)-1(x+2))} \\ &= \frac{2x+3}{x(x+2)(x-1)} \\ &\equiv \frac{2 \times 0 + 3}{x(0+2)(0-1)} \\ &\quad + \frac{2 \times (-2) + 3}{(-2)(x+2)(-2-1)} + \frac{2 \times 1 + 3}{1(1+2)(x-1)} \end{aligned}$$

Problem 2(v):

$$\int \frac{2x+3}{x^3+x^2-2x} dx$$

Solution:

Step 1: Partial Fraction

$$\begin{aligned} \frac{2x+3}{x^3+x^2-2x} &= \frac{2x+3}{x(x^2+x-2)} \\ &= \frac{2x+3}{x(x^2+2x-x-2)} \\ &= \frac{2x+3}{x(x(x+2)-1(x+2))} \\ &= \frac{2x+3}{x(x+2)(x-1)} \\ &\equiv \frac{2 \times 0 + 3}{x(0+2)(0-1)} \\ &\quad + \frac{2 \times (-2) + 3}{(-2)(x+2)(-2-1)} + \frac{2 \times 1 + 3}{1(1+2)(x-1)} \end{aligned}$$

$$\frac{2x+3}{x^3+x^2-2x} = \frac{-\frac{3}{2}}{x} + \frac{\frac{1}{6}}{x+2} + \frac{\frac{5}{3}}{x-1}$$

Step 2: Integration

$$\text{Let, } I = \int \frac{2x+3}{x^3+x^2-2x} dx$$

Problem 2(v):

$$\int \frac{2x+3}{x^3+x^2-2x} dx$$

Solution:

Step 1: Partial Fraction

$$\begin{aligned} \frac{2x+3}{x^3+x^2-2x} &= \frac{2x+3}{x(x^2+x-2)} \\ &= \frac{2x+3}{x(x^2+2x-x-2)} \\ &= \frac{2x+3}{x(x(x+2)-1(x+2))} \\ &= \frac{2x+3}{x(x+2)(x-1)} \\ &\equiv \frac{2 \times 0 + 3}{x(0+2)(0-1)} \\ &\quad + \frac{2 \times (-2) + 3}{(-2)(x+2)(-2-1)} + \frac{2 \times 1 + 3}{1(1+2)(x-1)} \end{aligned}$$

$$\frac{2x+3}{x^3+x^2-2x} = \frac{-\frac{3}{2}}{x} + \frac{\frac{1}{6}}{x+2} + \frac{\frac{5}{3}}{x-1}$$

Step 2: Integration

$$\begin{aligned} \text{Let, } I &= \int \frac{2x+3}{x^3+x^2-2x} dx \\ &= \int \left(\frac{-\frac{3}{2}}{x} + \frac{\frac{1}{6}}{x+2} + \frac{\frac{5}{3}}{x-1} \right) dx \end{aligned}$$

Problem 2(v):

$$\int \frac{2x+3}{x^3+x^2-2x} dx$$

Solution:

Step 1: Partial Fraction

$$\begin{aligned} \frac{2x+3}{x^3+x^2-2x} &= \frac{2x+3}{x(x^2+x-2)} \\ &= \frac{2x+3}{x(x^2+2x-x-2)} \\ &= \frac{2x+3}{x(x(x+2)-1(x+2))} \\ &= \frac{2x+3}{x(x+2)(x-1)} \\ &\equiv \frac{2 \times 0 + 3}{x(0+2)(0-1)} \\ &\quad + \frac{2 \times (-2) + 3}{(-2)(x+2)(-2-1)} + \frac{2 \times 1 + 3}{1(1+2)(x-1)} \end{aligned}$$

$$\frac{2x+3}{x^3+x^2-2x} = \frac{-\frac{3}{2}}{x} + \frac{\frac{1}{6}}{x+2} + \frac{\frac{5}{3}}{x-1}$$

Step 2: Integration

$$\begin{aligned} \text{Let, } I &= \int \frac{2x+3}{x^3+x^2-2x} dx \\ &= \int \left(\frac{-\frac{3}{2}}{x} + \frac{\frac{1}{6}}{x+2} + \frac{\frac{5}{3}}{x-1} \right) dx \\ &= \int \frac{-\frac{3}{2}}{x} dx + \int \frac{\frac{1}{6}}{x+2} dx + \int \frac{\frac{5}{3}}{x-1} dx \end{aligned}$$

Problem 2(v):

$$\int \frac{2x+3}{x^3+x^2-2x} dx$$

Solution:

Step 1: Partial Fraction

$$\begin{aligned} \frac{2x+3}{x^3+x^2-2x} &= \frac{2x+3}{x(x^2+x-2)} \\ &= \frac{2x+3}{x(x^2+2x-x-2)} \\ &= \frac{2x+3}{x(x(x+2)-1(x+2))} \\ &= \frac{2x+3}{x(x+2)(x-1)} \\ &\equiv \frac{2 \times 0 + 3}{x(0+2)(0-1)} \\ &\quad + \frac{2 \times (-2) + 3}{(-2)(x+2)(-2-1)} + \frac{2 \times 1 + 3}{1(1+2)(x-1)} \end{aligned}$$

$$\frac{2x+3}{x^3+x^2-2x} = \frac{-\frac{3}{2}}{x} + \frac{\frac{1}{6}}{x+2} + \frac{\frac{5}{3}}{x-1}$$

Step 2: Integration

$$\begin{aligned} \text{Let, } I &= \int \frac{2x+3}{x^3+x^2-2x} dx \\ &= \int \left(\frac{-\frac{3}{2}}{x} + \frac{\frac{1}{6}}{x+2} + \frac{\frac{5}{3}}{x-1} \right) dx \\ &= \int \frac{-\frac{3}{2}}{x} dx + \int \frac{\frac{1}{6}}{x+2} dx + \int \frac{\frac{5}{3}}{x-1} dx \\ &= \frac{-3}{2} \int \frac{1}{x} dx + \frac{1}{6} \int \frac{1}{x+2} dx + \frac{5}{3} \int \frac{1}{x-1} dx \end{aligned}$$

Problem 2(v):

$$\int \frac{2x+3}{x^3+x^2-2x} dx$$

Solution:

Step 1: Partial Fraction

$$\begin{aligned} \frac{2x+3}{x^3+x^2-2x} &= \frac{2x+3}{x(x^2+x-2)} \\ &= \frac{2x+3}{x(x^2+2x-x-2)} \\ &= \frac{2x+3}{x(x(x+2)-1(x+2))} \\ &= \frac{2x+3}{x(x+2)(x-1)} \\ &\equiv \frac{2 \times 0 + 3}{x(0+2)(0-1)} \\ &\quad + \frac{2 \times (-2) + 3}{(-2)(x+2)(-2-1)} + \frac{2 \times 1 + 3}{1(1+2)(x-1)} \end{aligned}$$

$$\frac{2x+3}{x^3+x^2-2x} = \frac{-\frac{3}{2}}{x} + \frac{\frac{1}{6}}{x+2} + \frac{\frac{5}{3}}{x-1}$$

Step 2: Integration

$$\begin{aligned} \text{Let, } I &= \int \frac{2x+3}{x^3+x^2-2x} dx \\ &= \int \left(\frac{-\frac{3}{2}}{x} + \frac{\frac{1}{6}}{x+2} + \frac{\frac{5}{3}}{x-1} \right) dx \\ &= \int \frac{-\frac{3}{2}}{x} dx + \int \frac{\frac{1}{6}}{x+2} dx + \int \frac{\frac{5}{3}}{x-1} dx \\ &= \frac{-3}{2} \int \frac{1}{x} dx + \frac{1}{6} \int \frac{1}{x+2} dx + \frac{5}{3} \int \frac{1}{x-1} dx \\ &= \frac{-3}{2} \ln x + \frac{1}{6} \ln(x+2) + \frac{5}{3} \ln(x-1) + c \end{aligned}$$

Exercise 10.4

Problem 3(iii):

$$\int \frac{x}{(x-1)^2(x+2)} dx$$

Solution:

Exercise 10.4

Problem 3(iii):

$$\int \frac{x}{(x-1)^2(x+2)} dx$$

Solution:

Step 1: Partial Fraction

Exercise 10.4

Problem 3(iii):

$$\int \frac{x}{(x-1)^2(x+2)} dx$$

Solution:

Step 1: Partial Fraction

$$\frac{x}{(x-1)^2(x+2)} \equiv \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2} \dots (1)$$

Exercise 10.4

Problem 3(iii):

$$\int \frac{x}{(x-1)^2(x+2)} dx$$

Solution:

Step 1: Partial Fraction

$$\frac{x}{(x-1)^2(x+2)} \equiv \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2} \dots (1)$$

Multiplying both side of (1) by $(x-1)^2$ and taking limit $x \rightarrow 1$,

Problem 3(iii):

$$\int \frac{x}{(x-1)^2(x+2)} dx$$

Solution:

Step 1: Partial Fraction

$$\frac{x}{(x-1)^2(x+2)} \equiv \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2} \dots (1)$$

Multipling both side of (1) by $(x-1)^2$ and taking
limit $x \rightarrow 1$,

$$\begin{aligned} \lim_{x \rightarrow 1} \left[\frac{x(x-1)^2}{(x-1)^2(x+2)} \right] &= \lim_{x \rightarrow 1} \left[\frac{A(x-1)^2}{(x-1)} \right] \\ &+ \lim_{x \rightarrow 1} \left[\frac{B(x-1)^2}{(x-1)^2} \right] \\ &+ \lim_{x \rightarrow 1} \left[\frac{C(x-1)^2}{x+2} \right] \end{aligned}$$

Problem 3(iii):

$$\int \frac{x}{(x-1)^2(x+2)} dx$$

Solution:

Step 1: Partial Fraction

$$\frac{x}{(x-1)^2(x+2)} \equiv \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2} \dots (1)$$

Multiplying both side of (1) by $(x-1)^2$ and taking limit $x \rightarrow 1$,

$$\begin{aligned} \lim_{x \rightarrow 1} \left[\frac{x(x-1)^2}{(x-1)^2(x+2)} \right] &= \lim_{x \rightarrow 1} \left[\frac{A(x-1)^2}{(x-1)} \right] \\ &+ \lim_{x \rightarrow 1} \left[\frac{B(x-1)^2}{(x-1)^2} \right] \\ &+ \lim_{x \rightarrow 1} \left[\frac{C(x-1)^2}{x+2} \right] \end{aligned}$$

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow 1} \left[\frac{x}{x+2} \right] &= \lim_{x \rightarrow 1} [A(x-1)] + \lim_{x \rightarrow 1} [B] \\ &+ \lim_{x \rightarrow 1} \left[\frac{C(x-1)^2}{x+2} \right] \end{aligned}$$

Problem 3(iii):

$$\int \frac{x}{(x-1)^2(x+2)} dx$$

Solution:

Step 1: Partial Fraction

$$\frac{x}{(x-1)^2(x+2)} \equiv \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2} \dots (1)$$

Multiplying both side of (1) by $(x-1)^2$ and taking limit $x \rightarrow 1$,

$$\begin{aligned} \lim_{x \rightarrow 1} \left[\frac{x(x-1)^2}{(x-1)^2(x+2)} \right] &= \lim_{x \rightarrow 1} \left[\frac{A(x-1)^2}{(x-1)} \right] \\ &+ \lim_{x \rightarrow 1} \left[\frac{B(x-1)^2}{(x-1)^2} \right] \\ &+ \lim_{x \rightarrow 1} \left[\frac{C(x-1)^2}{x+2} \right] \end{aligned}$$

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow 1} \left[\frac{x}{x+2} \right] &= \lim_{x \rightarrow 1} [A(x-1)] + \lim_{x \rightarrow 1} [B] \\ &+ \lim_{x \rightarrow 1} \left[\frac{C(x-1)^2}{x+2} \right] \end{aligned}$$

$$\Rightarrow \frac{1}{1+2} = B$$

Problem 3(iii):

$$\int \frac{x}{(x-1)^2(x+2)} dx$$

Solution:

Step 1: Partial Fraction

$$\frac{x}{(x-1)^2(x+2)} \equiv \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2} \dots (1)$$

Multiplying both side of (1) by $(x-1)^2$ and taking limit $x \rightarrow 1$,

$$\begin{aligned} \lim_{x \rightarrow 1} \left[\frac{x(x-1)^2}{(x-1)^2(x+2)} \right] &= \lim_{x \rightarrow 1} \left[\frac{A(x-1)^2}{(x-1)} \right] \\ &+ \lim_{x \rightarrow 1} \left[\frac{B(x-1)^2}{(x-1)^2} \right] \\ &+ \lim_{x \rightarrow 1} \left[\frac{C(x-1)^2}{x+2} \right] \end{aligned}$$

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow 1} \left[\frac{x}{x+2} \right] &= \lim_{x \rightarrow 1} [A(x-1)] + \lim_{x \rightarrow 1} [B] \\ &+ \lim_{x \rightarrow 1} \left[\frac{C(x-1)^2}{x+2} \right] \end{aligned}$$

$$\begin{aligned} \Rightarrow \frac{1}{1+2} &= B \\ \therefore B &= \frac{1}{3} \end{aligned}$$

Exercise 10.4

Problem 3(iii):

$$\int \frac{x}{(x-1)^2(x+2)} dx$$

Solution:

Step 1: Partial Fraction

$$\frac{x}{(x-1)^2(x+2)} \equiv \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2} \dots (1)$$

Multiplying both side of (1) by $(x-1)^2$ and taking limit $x \rightarrow 1$,

$$\begin{aligned} \lim_{x \rightarrow 1} \left[\frac{x(x-1)^2}{(x-1)^2(x+2)} \right] &= \lim_{x \rightarrow 1} \left[\frac{A(x-1)^2}{(x-1)} \right] \\ &+ \lim_{x \rightarrow 1} \left[\frac{B(x-1)^2}{(x-1)^2} \right] \\ &+ \lim_{x \rightarrow 1} \left[\frac{C(x-1)^2}{x+2} \right] \end{aligned}$$

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow 1} \left[\frac{x}{x+2} \right] &= \lim_{x \rightarrow 1} [A(x-1)] + \lim_{x \rightarrow 1} [B] \\ &+ \lim_{x \rightarrow 1} \left[\frac{C(x-1)^2}{x+2} \right] \end{aligned}$$

$$\begin{aligned} \Rightarrow \frac{1}{1+2} &= B \\ \therefore B &= \frac{1}{3} \end{aligned}$$

Short Method:(Applicable for B and C only)

Exercise 10.4

Problem 3(iii):

$$\int \frac{x}{(x-1)^2(x+2)} dx$$

Solution:

Step 1: Partial Fraction

$$\frac{x}{(x-1)^2(x+2)} \equiv \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2} \dots (1)$$

Multiplying both side of (1) by $(x-1)^2$ and taking limit $x \rightarrow 1$,

$$\begin{aligned} \lim_{x \rightarrow 1} \left[\frac{x(x-1)^2}{(x-1)^2(x+2)} \right] &= \lim_{x \rightarrow 1} \left[\frac{A(x-1)^2}{(x-1)} \right] \\ &+ \lim_{x \rightarrow 1} \left[\frac{B(x-1)^2}{(x-1)^2} \right] \\ &+ \lim_{x \rightarrow 1} \left[\frac{C(x-1)^2}{x+2} \right] \end{aligned}$$

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow 1} \left[\frac{x}{x+2} \right] &= \lim_{x \rightarrow 1} [A(x-1)] + \lim_{x \rightarrow 1} [B] \\ &+ \lim_{x \rightarrow 1} \left[\frac{C(x-1)^2}{x+2} \right] \end{aligned}$$

$$\begin{aligned} \Rightarrow \frac{1}{1+2} &= B \\ \therefore B &= \frac{1}{3} \end{aligned}$$

Short Method:(Applicable for B and C only)

$$B = \frac{1}{(x-1)^2(1+2)} = \frac{1}{3}$$

Problem 3(iii):

$$\int \frac{x}{(x-1)^2(x+2)} dx$$

Solution:

Step 1: Partial Fraction

$$\frac{x}{(x-1)^2(x+2)} \equiv \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2} \dots (1)$$

Multiplying both side of (1) by $(x-1)^2$ and taking limit $x \rightarrow 1$,

$$\begin{aligned} \lim_{x \rightarrow 1} \left[\frac{x(x-1)^2}{(x-1)^2(x+2)} \right] &= \lim_{x \rightarrow 1} \left[\frac{A(x-1)^2}{(x-1)} \right] \\ &+ \lim_{x \rightarrow 1} \left[\frac{B(x-1)^2}{(x-1)^2} \right] \\ &+ \lim_{x \rightarrow 1} \left[\frac{C(x-1)^2}{x+2} \right] \end{aligned}$$

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow 1} \left[\frac{x}{x+2} \right] &= \lim_{x \rightarrow 1} [A(x-1)] + \lim_{x \rightarrow 1} [B] \\ &+ \lim_{x \rightarrow 1} \left[\frac{C(x-1)^2}{x+2} \right] \end{aligned}$$

$$\begin{aligned} \Rightarrow \frac{1}{1+2} &= B \\ \therefore B &= \frac{1}{3} \end{aligned}$$

Short Method:(Applicable for B and C only)

$$\begin{aligned} B &= \frac{1}{(x-1)^2(1+2)} = \frac{1}{3} \\ C &= \frac{-2}{(-2-1)^2(x+2)} = \frac{-2}{9} \end{aligned}$$

Exercise 10.4

Problem 3(iii):

$$\int \frac{x}{(x-1)^2(x+2)} dx$$

Solution:

Step 1: Partial Fraction

$$\frac{x}{(x-1)^2(x+2)} \equiv \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2} \dots (1)$$

Multiplying both side of (1) by $(x-1)^2$ and taking limit $x \rightarrow 1$,

$$\begin{aligned} \lim_{x \rightarrow 1} \left[\frac{x(x-1)^2}{(x-1)^2(x+2)} \right] &= \lim_{x \rightarrow 1} \left[\frac{A(x-1)^2}{(x-1)} \right] \\ &+ \lim_{x \rightarrow 1} \left[\frac{B(x-1)^2}{(x-1)^2} \right] \\ &+ \lim_{x \rightarrow 1} \left[\frac{C(x-1)^2}{x+2} \right] \end{aligned}$$

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow 1} \left[\frac{x}{x+2} \right] &= \lim_{x \rightarrow 1} [A(x-1)] + \lim_{x \rightarrow 1} [B] \\ &+ \lim_{x \rightarrow 1} \left[\frac{C(x-1)^2}{x+2} \right] \end{aligned}$$

$$\begin{aligned} \Rightarrow \frac{1}{1+2} &= B \\ \therefore B &= \frac{1}{3} \end{aligned}$$

Short Method:(Applicable for B and C only)

$$\begin{aligned} B &= \frac{1}{(x-1)^2(1+2)} = \frac{1}{3} \\ C &= \frac{-2}{(-2-1)^2(x+2)} = \frac{-2}{9} \end{aligned}$$

Putting $x = 0$ on the both side of (1),

Exercise 10.4

Problem 3(iii):

$$\int \frac{x}{(x-1)^2(x+2)} dx$$

Solution:

Step 1: Partial Fraction

$$\frac{x}{(x-1)^2(x+2)} \equiv \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2} \dots (1)$$

Multiplying both side of (1) by $(x-1)^2$ and taking limit $x \rightarrow 1$,

$$\begin{aligned} \lim_{x \rightarrow 1} \left[\frac{x(x-1)^2}{(x-1)^2(x+2)} \right] &= \lim_{x \rightarrow 1} \left[\frac{A(x-1)^2}{(x-1)} \right] \\ &+ \lim_{x \rightarrow 1} \left[\frac{B(x-1)^2}{(x-1)^2} \right] \\ &+ \lim_{x \rightarrow 1} \left[\frac{C(x-1)^2}{x+2} \right] \end{aligned}$$

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow 1} \left[\frac{x}{x+2} \right] &= \lim_{x \rightarrow 1} [A(x-1)] + \lim_{x \rightarrow 1} [B] \\ &+ \lim_{x \rightarrow 1} \left[\frac{C(x-1)^2}{x+2} \right] \end{aligned}$$

$$\begin{aligned} \Rightarrow \frac{1}{1+2} &= B \\ \therefore B &= \frac{1}{3} \end{aligned}$$

Short Method:(Applicable for B and C only)

$$\begin{aligned} B &= \frac{1}{(x-1)^2(1+2)} = \frac{1}{3} \\ C &= \frac{-2}{(-2-1)^2(x+2)} = \frac{-2}{9} \end{aligned}$$

Putting $x = 0$ on the both side of (1),

$$0 = A + B + \frac{C}{2} \quad \therefore A = -\frac{2}{9}$$

Step 2: Integration

Step 2: Integration

$$\text{Let, } I = \int \frac{x}{(x-1)^2(x+2)} dx$$

Step 2: Integration

$$\begin{aligned}\text{Let, } I &= \int \frac{x}{(x-1)^2(x+2)} dx \\ &= \int \left(\frac{-\frac{2}{9}}{x-1} + \frac{\frac{1}{3}}{(x-1)^2} + \frac{-\frac{2}{9}}{x+2} \right) dx\end{aligned}$$

Step 2: Integration

$$\begin{aligned}\text{Let, } I &= \int \frac{x}{(x-1)^2(x+2)} dx \\ &= \int \left(\frac{-\frac{2}{9}}{x-1} + \frac{\frac{1}{3}}{(x-1)^2} + \frac{-\frac{2}{9}}{x+2} \right) dx \\ &= -\frac{2}{9} \int \frac{1}{x-1} dx + \frac{1}{3} \int \frac{1}{(x-1)^2} dx - \frac{2}{9} \int \frac{1}{x+2} dx\end{aligned}$$

Step 2: Integration

$$\begin{aligned}\text{Let, } I &= \int \frac{x}{(x-1)^2(x+2)} dx \\&= \int \left(\frac{-\frac{2}{9}}{x-1} + \frac{\frac{1}{3}}{(x-1)^2} + \frac{-\frac{2}{9}}{x+2} \right) dx \\&= -\frac{2}{9} \int \frac{1}{x-1} dx + \frac{1}{3} \int \frac{1}{(x-1)^2} dx - \frac{2}{9} \int \frac{1}{x+2} dx \\&= -\frac{2}{9} \ln(x-1) + \frac{1}{3} \frac{(x-1)^{-2+1}}{-2+1} - \frac{2}{9} \ln(x+2) + C\end{aligned}$$

Step 2: Integration

$$\begin{aligned}\text{Let, } I &= \int \frac{x}{(x-1)^2(x+2)} dx \\&= \int \left(\frac{-\frac{2}{9}}{x-1} + \frac{\frac{1}{3}}{(x-1)^2} + \frac{-\frac{2}{9}}{x+2} \right) dx \\&= -\frac{2}{9} \int \frac{1}{x-1} dx + \frac{1}{3} \int \frac{1}{(x-1)^2} dx - \frac{2}{9} \int \frac{1}{x+2} dx \\&= -\frac{2}{9} \ln(x-1) + \frac{1}{3} \frac{(x-1)^{-2+1}}{-2+1} - \frac{2}{9} \ln(x+2) + C \\&= -\frac{2}{9} \ln(x-1) - \frac{1}{3} \frac{1}{x-1} - \frac{2}{9} \ln(x+2) + C\end{aligned}$$

Problem 4(iii) :

$$\int \frac{x+2}{(1-x)(x^2+4)} dx$$

Exercise 10.4

Problem 4(iii) :

$$\int \frac{x+2}{(1-x)(x^2+4)} dx$$

Solution:

Step1: Partial Fraction

Exercise 10.4

Problem 4(iii) :

$$\int \frac{x+2}{(1-x)(x^2+4)} dx$$

Solution:

Step1: Partial Fraction

$$\frac{x+2}{(1-x)(x^2+4)} \equiv \frac{A}{1-x} + \frac{Bx+C}{x^2+4} \dots (1)$$

Exercise 10.4

Problem 4(iii) :

$$\int \frac{x+2}{(1-x)(x^2+4)} dx$$

Solution:

Step1: Partial Fraction

$$\frac{x+2}{(1-x)(x^2+4)} \equiv \frac{A}{1-x} + \frac{Bx+C}{x^2+4} \dots (1)$$

multiplying both side of (1) by $(1-x)$ and taking limit $x \rightarrow 1$,

Exercise 10.4

Problem 4(iii) :

$$\int \frac{x+2}{(1-x)(x^2+4)} dx$$

Solution:

Step1: Partial Fraction

$$\frac{x+2}{(1-x)(x^2+4)} \equiv \frac{A}{1-x} + \frac{Bx+C}{x^2+4} \dots (1)$$

multiplying both side of (1) by $(1-x)$ and taking limit $x \rightarrow 1$,

$$\begin{aligned} \lim_{x \rightarrow 1} \left[\frac{(x+2)(1-x)}{(1-x)(x^2+4)} \right] &\equiv \lim_{x \rightarrow 1} \left[\frac{A(1-x)}{(1-x)} \right] \\ &+ \lim_{x \rightarrow 1} \left[\frac{(Bx+c)(1-x)}{x^2+4} \right] \end{aligned}$$

Exercise 10.4

Problem 4(iii) :

$$\int \frac{x+2}{(1-x)(x^2+4)} dx$$

Solution:

Step1: Partial Fraction

$$\frac{x+2}{(1-x)(x^2+4)} \equiv \frac{A}{1-x} + \frac{Bx+C}{x^2+4} \dots (1)$$

multiplying both side of (1) by $(1-x)$ and taking limit $x \rightarrow 1$,

$$\begin{aligned} \lim_{x \rightarrow 1} \left[\frac{(x+2)(1-x)}{(1-x)(x^2+4)} \right] &\equiv \lim_{x \rightarrow 1} \left[\frac{A(1-x)}{(1-x)} \right] \\ &\quad + \lim_{x \rightarrow 1} \left[\frac{(Bx+c)(1-x)}{x^2+4} \right] \\ \Rightarrow \lim_{x \rightarrow 1} \left[\frac{x+2}{x^2+4} \right] &\equiv \lim_{x \rightarrow 1} [A] + \lim_{x \rightarrow 1} \left[\frac{(Bx+c)(1-x)}{(x^2+4)} \right] \end{aligned}$$

Exercise 10.4

Problem 4(iii) :

$$\int \frac{x+2}{(1-x)(x^2+4)} dx$$

$$\Rightarrow \frac{1+2}{1^2+4} \equiv A+0$$

$$\therefore A = \frac{3}{5}$$

Solution:

Step1: Partial Fraction

$$\frac{x+2}{(1-x)(x^2+4)} \equiv \frac{A}{1-x} + \frac{Bx+C}{x^2+4} \dots (1)$$

multiplying both side of (1) by $(1-x)$ and taking limit $x \rightarrow 1$,

$$\begin{aligned} \lim_{x \rightarrow 1} \left[\frac{(x+2)(1-x)}{(1-x)(x^2+4)} \right] &\equiv \lim_{x \rightarrow 1} \left[\frac{A(1-x)}{(1-x)} \right] \\ &\quad + \lim_{x \rightarrow 1} \left[\frac{(Bx+C)(1-x)}{x^2+4} \right] \\ \Rightarrow \lim_{x \rightarrow 1} \left[\frac{x+2}{x^2+4} \right] &\equiv \lim_{x \rightarrow 1} [A] + \lim_{x \rightarrow 1} \left[\frac{(Bx+C)(1-x)}{(x^2+4)} \right] \end{aligned}$$

Exercise 10.4

Problem 4(iii) :

$$\int \frac{x+2}{(1-x)(x^2+4)} dx$$

Solution:

Step1: Partial Fraction

$$\frac{x+2}{(1-x)(x^2+4)} \equiv \frac{A}{1-x} + \frac{Bx+C}{x^2+4} \dots (1)$$

multiplying both side of (1) by $(1-x)$ and taking limit $x \rightarrow 1$,

$$\begin{aligned} \lim_{x \rightarrow 1} \left[\frac{(x+2)(1-x)}{(1-x)(x^2+4)} \right] &\equiv \lim_{x \rightarrow 1} \left[\frac{A(1-x)}{(1-x)} \right] \\ &\quad + \lim_{x \rightarrow 1} \left[\frac{(Bx+c)(1-x)}{x^2+4} \right] \\ \Rightarrow \lim_{x \rightarrow 1} \left[\frac{x+2}{x^2+4} \right] &\equiv \lim_{x \rightarrow 1} [A] + \lim_{x \rightarrow 1} \left[\frac{(Bx+c)(1-x)}{(x^2+4)} \right] \end{aligned}$$

$$\begin{aligned} \Rightarrow \frac{1+2}{1^2+4} &\equiv A+0 \\ \therefore A &= \frac{3}{5} \end{aligned}$$

Now, multiplying both side of (1) by x and taking limit $x \rightarrow \infty$,

Exercise 10.4

Problem 4(iii) :

$$\int \frac{x+2}{(1-x)(x^2+4)} dx$$

Solution:

Step1: Partial Fraction

$$\frac{x+2}{(1-x)(x^2+4)} \equiv \frac{A}{1-x} + \frac{Bx+C}{x^2+4} \dots (1)$$

multiplying both side of (1) by $(1-x)$ and taking limit $x \rightarrow 1$,

$$\begin{aligned} \lim_{x \rightarrow 1} \left[\frac{(x+2)(1-x)}{(1-x)(x^2+4)} \right] &\equiv \lim_{x \rightarrow 1} \left[\frac{A(1-x)}{(1-x)} \right] \\ &+ \lim_{x \rightarrow 1} \left[\frac{(Bx+C)(1-x)}{x^2+4} \right] \\ \Rightarrow \lim_{x \rightarrow 1} \left[\frac{x+2}{x^2+4} \right] &\equiv \lim_{x \rightarrow 1} [A] + \lim_{x \rightarrow 1} \left[\frac{(Bx+C)(1-x)}{(x^2+4)} \right] \end{aligned}$$

$$\Rightarrow \frac{1+2}{1^2+4} \equiv A+0$$

$$\therefore A = \frac{3}{5}$$

Now, multiplying both side of (1) by x and taking limit $x \rightarrow \infty$,

$$\begin{aligned} \lim_{x \rightarrow \infty} \left[\frac{(x+2)x}{(1-x)(x^2+4)} \right] &\equiv \lim_{x \rightarrow \infty} \left[\frac{Ax}{1-x} \right] \\ &+ \lim_{x \rightarrow \infty} \left[\frac{(Bx^2+cx)}{x^2+4} \right] \end{aligned}$$

Exercise 10.4

Problem 4(iii) :

$$\int \frac{x+2}{(1-x)(x^2+4)} dx$$

Solution:

Step1: Partial Fraction

$$\frac{x+2}{(1-x)(x^2+4)} \equiv \frac{A}{1-x} + \frac{Bx+C}{x^2+4} \dots (1)$$

multiplying both side of (1) by $(1-x)$ and taking limit $x \rightarrow 1$,

$$\begin{aligned} \lim_{x \rightarrow 1} \left[\frac{(x+2)(1-x)}{(1-x)(x^2+4)} \right] &\equiv \lim_{x \rightarrow 1} \left[\frac{A(1-x)}{(1-x)} \right] \\ &\quad + \lim_{x \rightarrow 1} \left[\frac{(Bx+C)(1-x)}{x^2+4} \right] \\ \Rightarrow \lim_{x \rightarrow 1} \left[\frac{x+2}{x^2+4} \right] &\equiv \lim_{x \rightarrow 1} [A] + \lim_{x \rightarrow 1} \left[\frac{(Bx+C)(1-x)}{(x^2+4)} \right] \end{aligned}$$

$$\begin{aligned} \Rightarrow \frac{1+2}{1^2+4} &\equiv A+0 \\ \therefore A &= \frac{3}{5} \end{aligned}$$

Now, multiplying both side of (1) by x and taking limit $x \rightarrow \infty$,

$$\begin{aligned} \lim_{x \rightarrow \infty} \left[\frac{(x+2)x}{(1-x)(x^2+4)} \right] &\equiv \lim_{x \rightarrow \infty} \left[\frac{Ax}{1-x} \right] \\ &\quad + \lim_{x \rightarrow \infty} \left[\frac{(Bx^2+cx)}{x^2+4} \right] \\ \Rightarrow 0 &\equiv -A+B \\ \therefore A=B &= \frac{3}{5} \end{aligned}$$

Exercise 10.4

Problem 4(iii) :

$$\int \frac{x+2}{(1-x)(x^2+4)} dx$$

Solution:

Step1: Partial Fraction

$$\frac{x+2}{(1-x)(x^2+4)} \equiv \frac{A}{1-x} + \frac{Bx+C}{x^2+4} \dots (1)$$

multiplying both side of (1) by $(1-x)$ and taking limit $x \rightarrow 1$,

$$\begin{aligned} \lim_{x \rightarrow 1} \left[\frac{(x+2)(1-x)}{(1-x)(x^2+4)} \right] &\equiv \lim_{x \rightarrow 1} \left[\frac{A(1-x)}{(1-x)} \right] \\ &\quad + \lim_{x \rightarrow 1} \left[\frac{(Bx+C)(1-x)}{x^2+4} \right] \\ \Rightarrow \lim_{x \rightarrow 1} \left[\frac{x+2}{x^2+4} \right] &\equiv \lim_{x \rightarrow 1} [A] + \lim_{x \rightarrow 1} \left[\frac{(Bx+C)(1-x)}{(x^2+4)} \right] \end{aligned}$$

$$\Rightarrow \frac{1+2}{1^2+4} \equiv A+0$$

$$\therefore A = \frac{3}{5}$$

Now, multiplying both side of (1) by x and taking limit $x \rightarrow \infty$,

$$\begin{aligned} \lim_{x \rightarrow \infty} \left[\frac{(x+2)x}{(1-x)(x^2+4)} \right] &\equiv \lim_{x \rightarrow \infty} \left[\frac{Ax}{1-x} \right] \\ &\quad + \lim_{x \rightarrow \infty} \left[\frac{(Bx^2+cx)}{x^2+4} \right] \\ \Rightarrow 0 &\equiv -A+B \end{aligned}$$

$$\therefore A = B = \frac{3}{5}$$

Note: You can use short method (See Problem 1(vii)) for finding 'A'

Exercise 10.4

Again, put $x = 0$ in (1),

$$\frac{0 + 2}{(1 - 0)(0^2 + 4)} = \frac{A}{1 - 0} + \frac{B \times 0 + C}{0^2 + 4}$$

Exercise 10.4

Again, put $x = 0$ in (1),

$$\begin{aligned}\frac{0+2}{(1-0)(0^2+4)} &= \frac{A}{1-0} + \frac{B \times 0 + C}{0^2+4} \\ \Rightarrow \frac{1}{2} &= A + \frac{C}{4}\end{aligned}$$

Exercise 10.4

Again, put $x = 0$ in (1),

$$\begin{aligned}\frac{0+2}{(1-0)(0^2+4)} &= \frac{A}{1-0} + \frac{B \times 0 + C}{0^2+4} \\ \Rightarrow \frac{1}{2} &= A + \frac{C}{4} \\ \Rightarrow \frac{1}{2} &= \frac{3}{5} + \frac{C}{4}\end{aligned}$$

Exercise 10.4

Again, put $x = 0$ in (1),

$$\begin{aligned}\frac{0+2}{(1-0)(0^2+4)} &= \frac{A}{1-0} + \frac{B \times 0 + C}{0^2+4} \\ \Rightarrow \frac{1}{2} &= A + \frac{C}{4} \\ \Rightarrow \frac{1}{2} &= \frac{3}{5} + \frac{C}{4} \\ \therefore C &= \frac{-2}{5}\end{aligned}$$

Exercise 10.4

Again, put $x = 0$ in (1),

$$\begin{aligned}\frac{0+2}{(1-0)(0^2+4)} &= \frac{A}{1-0} + \frac{B \times 0 + C}{0^2+4} \\ \Rightarrow \frac{1}{2} &= A + \frac{C}{4} \\ \Rightarrow \frac{1}{2} &= \frac{3}{5} + \frac{C}{4} \\ \therefore C &= \frac{-2}{5}\end{aligned}$$

Step 2: Integration

Exercise 10.4

Again, put $x = 0$ in (1),

$$\begin{aligned}\frac{0+2}{(1-0)(0^2+4)} &= \frac{A}{1-0} + \frac{B \times 0 + C}{0^2+4} \\ \Rightarrow \frac{1}{2} &= A + \frac{C}{4} \\ \Rightarrow \frac{1}{2} &= \frac{3}{5} + \frac{C}{4} \\ \therefore C &= \frac{-2}{5}\end{aligned}$$

Step 2: Integration

$$\text{Let, } I = \int \frac{x+2}{(1-x)(x^2+4)} dx$$

Exercise 10.4

Again, put $x = 0$ in (1),

$$\begin{aligned}\frac{0+2}{(1-0)(0^2+4)} &= \frac{A}{1-0} + \frac{B \times 0 + C}{0^2+4} \\ \Rightarrow \frac{1}{2} &= A + \frac{C}{4} \\ \Rightarrow \frac{1}{2} &= \frac{3}{5} + \frac{C}{4} \\ \therefore C &= \frac{-2}{5}\end{aligned}$$

Step 2: Integration

$$\begin{aligned}\text{Let, } I &= \int \frac{x+2}{(1-x)(x^2+4)} dx \\ &= \int \left(\frac{\frac{3}{5}}{1-x} + \frac{\frac{3}{5}x - \frac{2}{5}}{x^2+4} \right) dx\end{aligned}$$

Exercise 10.4

Again, put $x = 0$ in (1),

$$\begin{aligned}\frac{0+2}{(1-0)(0^2+4)} &= \frac{A}{1-0} + \frac{B \times 0 + C}{0^2+4} \\ \Rightarrow \frac{1}{2} &= A + \frac{C}{4} \\ \Rightarrow \frac{1}{2} &= \frac{3}{5} + \frac{C}{4} \\ \therefore C &= \frac{-2}{5}\end{aligned}$$

Step 2: Integration

$$\begin{aligned}\text{Let, } I &= \int \frac{x+2}{(1-x)(x^2+4)} dx \\ &= \int \left(\frac{\frac{3}{5}}{1-x} + \frac{\frac{3}{5}x - \frac{2}{5}}{x^2+4} \right) dx \\ &= \int \frac{\frac{3}{5}}{1-x} dx + \int \frac{\frac{3}{5}x - \frac{2}{5}}{x^2+4} dx\end{aligned}$$

Exercise 10.4

Again, put $x = 0$ in (1),

$$\begin{aligned}\frac{0+2}{(1-0)(0^2+4)} &= \frac{A}{1-0} + \frac{B \times 0 + C}{0^2+4} \\ \Rightarrow \frac{1}{2} &= A + \frac{C}{4} \\ \Rightarrow \frac{1}{2} &= \frac{3}{5} + \frac{C}{4} \\ \therefore C &= \frac{-2}{5}\end{aligned}$$

$$= \frac{3}{5} \int \frac{1}{1-x} dx + \int \frac{\frac{3}{5}x}{x^2+4} dx + \int \frac{-\frac{2}{5}}{x^2+4} dx$$

Step 2: Integration

$$\begin{aligned}\text{Let, } I &= \int \frac{x+2}{(1-x)(x^2+4)} dx \\ &= \int \left(\frac{\frac{3}{5}}{1-x} + \frac{\frac{3}{5}x - \frac{2}{5}}{x^2+4} \right) dx \\ &= \int \frac{\frac{3}{5}}{1-x} dx + \int \frac{\frac{3}{5}x - \frac{2}{5}}{x^2+4} dx\end{aligned}$$

Exercise 10.4

Again, put $x = 0$ in (1),

$$\begin{aligned}\frac{0+2}{(1-0)(0^2+4)} &= \frac{A}{1-0} + \frac{B \times 0 + C}{0^2+4} \\ \Rightarrow \frac{1}{2} &= A + \frac{C}{4} \\ \Rightarrow \frac{1}{2} &= \frac{3}{5} + \frac{C}{4} \\ \therefore C &= \frac{-2}{5}\end{aligned}$$

$$\begin{aligned}&= \frac{3}{5} \int \frac{1}{1-x} dx + \int \frac{\frac{3}{5}x}{x^2+4} dx + \int \frac{-\frac{2}{5}}{x^2+4} dx \\ &= -\frac{3}{5} \ln(1-x) + \frac{3}{5} \int \frac{\frac{1}{2} \times 2x}{x^2+4} dx - \frac{2}{5} \int \frac{1}{x^2+2^2} dx \\ &= -\frac{3}{5} \ln(1-x) + \frac{3}{5} \times \frac{1}{2} \times \ln(x^2+4) - \frac{2}{5} \times \frac{1}{2} \arctan \frac{x}{2}\end{aligned}$$

Step 2: Integration

$$\begin{aligned}\text{Let, } I &= \int \frac{x+2}{(1-x)(x^2+4)} dx \\ &= \int \left(\frac{\frac{3}{5}}{1-x} + \frac{\frac{3}{5}x - \frac{2}{5}}{x^2+4} \right) dx \\ &= \int \frac{\frac{3}{5}}{1-x} dx + \int \frac{\frac{3}{5}x - \frac{2}{5}}{x^2+4} dx\end{aligned}$$

Exercise 10.4

Again, put $x = 0$ in (1),

$$\begin{aligned}\frac{0+2}{(1-0)(0^2+4)} &= \frac{A}{1-0} + \frac{B \times 0 + C}{0^2+4} \\ \Rightarrow \frac{1}{2} &= A + \frac{C}{4} \\ \Rightarrow \frac{1}{2} &= \frac{3}{5} + \frac{C}{4} \\ \therefore C &= \frac{-2}{5}\end{aligned}$$

Step 2: Integration

$$\begin{aligned}\text{Let, } I &= \int \frac{x+2}{(1-x)(x^2+4)} dx \\ &= \int \left(\frac{\frac{3}{5}}{1-x} + \frac{\frac{3}{5}x - \frac{2}{5}}{x^2+4} \right) dx \\ &= \int \frac{\frac{3}{5}}{1-x} dx + \int \frac{\frac{3}{5}x - \frac{2}{5}}{x^2+4} dx\end{aligned}$$

$$\begin{aligned}&= \frac{3}{5} \int \frac{1}{1-x} dx + \int \frac{\frac{3}{5}x}{x^2+4} dx + \int \frac{-\frac{2}{5}}{x^2+4} dx \\ &= -\frac{3}{5} \ln(1-x) + \frac{3}{5} \int \frac{\frac{1}{2} \times 2x}{x^2+4} dx - \frac{2}{5} \int \frac{1}{x^2+2^2} dx \\ &= -\frac{3}{5} \ln(1-x) + \frac{3}{5} \times \frac{1}{2} \times \ln(x^2+4) - \frac{2}{5} \times \frac{1}{2} \arctan \frac{x}{2} \\ &= -\frac{3}{5} \ln(1-x) + \frac{3}{10} \ln(x^2+4) - \frac{1}{5} \arctan \frac{x}{2} + c\end{aligned}$$

Exercise 10.4

Problem 5(i):

$$\int \frac{2x + 1}{(2x + 3)^2} dx$$

Solution:

Problem 5(i):

$$\int \frac{2x+1}{(2x+3)^2} dx$$

Solution:

Step 1: Partial Fraction

$$\frac{2x+1}{(2x+3)^2} \equiv \frac{A}{(2x+3)} + \frac{B}{(2x+3)^2} \dots (1)$$

Problem 5(i):

$$\int \frac{2x+1}{(2x+3)^2} dx$$

Solution:

Step 1: Partial Fraction

$$\frac{2x+1}{(2x+3)^2} \equiv \frac{A}{(2x+3)} + \frac{B}{(2x+3)^2} \dots (1)$$

Short Method:

Problem 5(i):

$$\int \frac{2x+1}{(2x+3)^2} dx$$

Solution:

Step 1: Partial Fraction

$$\frac{2x+1}{(2x+3)^2} \equiv \frac{A}{(2x+3)} + \frac{B}{(2x+3)^2} \dots (1)$$

Short Method:

$$B = \frac{2 \times \frac{-3}{2} + 1}{1} = -2$$

Problem 5(i):

$$\int \frac{2x+1}{(2x+3)^2} dx$$

Solution:

Step 1: Partial Fraction

$$\frac{2x+1}{(2x+3)^2} \equiv \frac{A}{(2x+3)} + \frac{B}{(2x+3)^2} \dots (1)$$

Short Method:

$$B = \frac{2 \times \frac{-3}{2} + 1}{1} = -2$$

Putting $x = 0$ in (1),

Problem 5(i):

$$\int \frac{2x+1}{(2x+3)^2} dx$$

Solution:

Step 1: Partial Fraction

$$\frac{2x+1}{(2x+3)^2} \equiv \frac{A}{(2x+3)} + \frac{B}{(2x+3)^2} \dots (1)$$

Short Method:

$$B = \frac{2 \times \frac{-3}{2} + 1}{1} = -2$$

Putting $x = 0$ in (1),

$$\frac{2 \times 0 + 1}{(2 \times 0 + 3)^2} = \frac{A}{2 \times 0 + 3} + \frac{-2}{(2 \times 0 + 3)^2}$$

Problem 5(i):

$$\int \frac{2x+1}{(2x+3)^2} dx$$

Solution:

Step 1: Partial Fraction

$$\frac{2x+1}{(2x+3)^2} \equiv \frac{A}{(2x+3)} + \frac{B}{(2x+3)^2} \dots (1)$$

Short Method:

$$B = \frac{2 \times \frac{-3}{2} + 1}{1} = -2$$

Putting $x = 0$ in (1),

$$\begin{aligned} \frac{2 \times 0 + 1}{(2 \times 0 + 3)^2} &= \frac{A}{2 \times 0 + 3} + \frac{-2}{(2 \times 0 + 3)^2} \\ \frac{1}{9} &= \frac{A}{3} - \frac{2}{9} \end{aligned}$$

Problem 5(i):

$$\int \frac{2x+1}{(2x+3)^2} dx$$

Solution:

Step 1: Partial Fraction

$$\frac{2x+1}{(2x+3)^2} \equiv \frac{A}{(2x+3)} + \frac{B}{(2x+3)^2} \dots (1)$$

Short Method:

$$B = \frac{2 \times \frac{-3}{2} + 1}{1} = -2$$

Putting $x = 0$ in (1),

$$\frac{2 \times 0 + 1}{(2 \times 0 + 3)^2} = \frac{A}{2 \times 0 + 3} + \frac{-2}{(2 \times 0 + 3)^2}$$

$$\frac{1}{9} = \frac{A}{3} - \frac{2}{9}$$

$$\therefore A = 1$$

Problem 5(i):

Step 2: Integration

$$\int \frac{2x+1}{(2x+3)^2} dx$$

Solution:

Step 1: Partial Fraction

$$\frac{2x+1}{(2x+3)^2} \equiv \frac{A}{(2x+3)} + \frac{B}{(2x+3)^2} \dots (1)$$

Short Method:

$$B = \frac{2 \times \frac{-3}{2} + 1}{1} = -2$$

Putting $x = 0$ in (1),

$$\frac{2 \times 0 + 1}{(2 \times 0 + 3)^2} = \frac{A}{2 \times 0 + 3} + \frac{-2}{(2 \times 0 + 3)^2}$$

$$\frac{1}{9} = \frac{A}{3} - \frac{2}{9}$$

$$\therefore A = 1$$

Problem 5(i):

$$\int \frac{2x+1}{(2x+3)^2} dx$$

Solution:

Step 1: Partial Fraction

$$\frac{2x+1}{(2x+3)^2} \equiv \frac{A}{(2x+3)} + \frac{B}{(2x+3)^2} \dots (1)$$

Short Method:

$$B = \frac{2 \times \frac{-3}{2} + 1}{1} = -2$$

Putting $x = 0$ in (1),

$$\frac{2 \times 0 + 1}{(2 \times 0 + 3)^2} = \frac{A}{2 \times 0 + 3} + \frac{-2}{(2 \times 0 + 3)^2}$$

$$\frac{1}{9} = \frac{A}{3} - \frac{2}{9}$$

$$\therefore A = 1$$

Step 2: Integration

$$\text{Let, } I = \int \frac{2x+1}{(2x+3)^2} dx$$

Problem 5(i):

$$\int \frac{2x+1}{(2x+3)^2} dx$$

Solution:

Step 1: Partial Fraction

$$\frac{2x+1}{(2x+3)^2} \equiv \frac{A}{(2x+3)} + \frac{B}{(2x+3)^2} \dots (1)$$

Short Method:

$$B = \frac{2 \times \frac{-3}{2} + 1}{1} = -2$$

Putting $x = 0$ in (1),

$$\frac{2 \times 0 + 1}{(2 \times 0 + 3)^2} = \frac{A}{2 \times 0 + 3} + \frac{-2}{(2 \times 0 + 3)^2}$$

$$\frac{1}{9} = \frac{A}{3} - \frac{2}{9}$$

$$\therefore A = 1$$

Step 2: Integration

$$\begin{aligned} \text{Let, } I &= \int \frac{2x+1}{(2x+3)^2} dx \\ &= \int \left(\frac{1}{(2x+3)} - \frac{2}{(2x+3)^2} \right) dx \end{aligned}$$

Problem 5(i):

$$\int \frac{2x+1}{(2x+3)^2} dx$$

Solution:

Step 1: Partial Fraction

$$\frac{2x+1}{(2x+3)^2} \equiv \frac{A}{(2x+3)} + \frac{B}{(2x+3)^2} \dots (1)$$

Short Method:

$$B = \frac{2 \times \frac{-3}{2} + 1}{1} = -2$$

Putting $x = 0$ in (1),

$$\frac{2 \times 0 + 1}{(2 \times 0 + 3)^2} = \frac{A}{2 \times 0 + 3} + \frac{-2}{(2 \times 0 + 3)^2}$$

$$\frac{1}{9} = \frac{A}{3} - \frac{2}{9}$$

$$\therefore A = 1$$

Step 2: Integration

$$\begin{aligned} \text{Let, } I &= \int \frac{2x+1}{(2x+3)^2} dx \\ &= \int \left(\frac{1}{(2x+3)} - \frac{2}{(2x+3)^2} \right) dx \\ &= \int \frac{1}{2x+3} dx - \int \frac{2}{(2x+3)^2} dx \end{aligned}$$

Exercise 10.4

Problem 5(i):

$$\int \frac{2x+1}{(2x+3)^2} dx$$

Solution:

Step 1: Partial Fraction

$$\frac{2x+1}{(2x+3)^2} \equiv \frac{A}{(2x+3)} + \frac{B}{(2x+3)^2} \dots (1)$$

Short Method:

$$B = \frac{2 \times \frac{-3}{2} + 1}{1} = -2$$

Putting $x = 0$ in (1),

$$\frac{2 \times 0 + 1}{(2 \times 0 + 3)^2} = \frac{A}{2 \times 0 + 3} + \frac{-2}{(2 \times 0 + 3)^2}$$

$$\frac{1}{9} = \frac{A}{3} - \frac{2}{9}$$

$$\therefore A = 1$$

Step 2: Integration

$$\begin{aligned} \text{Let, } I &= \int \frac{2x+1}{(2x+3)^2} dx \\ &= \int \left(\frac{1}{(2x+3)} - \frac{2}{(2x+3)^2} \right) dx \\ &= \int \frac{1}{2x+3} dx - \int \frac{2}{(2x+3)^2} dx \\ &= \frac{1}{2} \ln(2x+3) - 2 \times \frac{1}{2} \times \frac{(2x+3)^{-2+1}}{-2+1} \end{aligned}$$

Problem 5(i):

$$\int \frac{2x+1}{(2x+3)^2} dx$$

Solution:

Step 1: Partial Fraction

$$\frac{2x+1}{(2x+3)^2} \equiv \frac{A}{(2x+3)} + \frac{B}{(2x+3)^2} \dots (1)$$

Short Method:

$$B = \frac{2 \times \frac{-3}{2} + 1}{1} = -2$$

Putting $x = 0$ in (1),

$$\frac{2 \times 0 + 1}{(2 \times 0 + 3)^2} = \frac{A}{2 \times 0 + 3} + \frac{-2}{(2 \times 0 + 3)^2}$$

$$\frac{1}{9} = \frac{A}{3} - \frac{2}{9}$$

$$\therefore A = 1$$

Step 2: Integration

$$\begin{aligned} \text{Let, } I &= \int \frac{2x+1}{(2x+3)^2} dx \\ &= \int \left(\frac{1}{(2x+3)} - \frac{2}{(2x+3)^2} \right) dx \\ &= \int \frac{1}{2x+3} dx - \int \frac{2}{(2x+3)^2} dx \\ &= \frac{1}{2} \ln(2x+3) - 2 \times \frac{1}{2} \times \frac{(2x+3)^{-2+1}}{-2+1} \\ &= \frac{1}{2} \ln(2x+3) + \frac{1}{2x+3} + C \end{aligned}$$

Problem 6(iv):

$$\int \frac{x^2 - 1}{x^2 - 4} dx$$

Solution:

Exercise 10.4

Problem 6(iv):

$$\int \frac{x^2 - 1}{x^2 - 4} dx$$

Solution:

Step 1: Partial fraction

Problem 6(iv):

$$\int \frac{x^2 - 1}{x^2 - 4} dx$$

Solution:

Step 1: Partial fraction

$$\frac{x^2 - 1}{x^2 - 4} = \frac{x^2 - 4 + 3}{x^2 - 4}$$

Problem 6(iv):

$$\int \frac{x^2 - 1}{x^2 - 4} dx$$

Solution:

Step 1: Partial fraction

$$\begin{aligned}\frac{x^2 - 1}{x^2 - 4} &= \frac{x^2 - 4 + 3}{x^2 - 4} \\ &= \frac{x^2 - 4}{x^2 - 4} + \frac{3}{x^2 - 4}\end{aligned}$$

Problem 6(iv):

$$\int \frac{x^2 - 1}{x^2 - 4} dx$$

Solution:

Step 1: Partial fraction

$$\begin{aligned}\frac{x^2 - 1}{x^2 - 4} &= \frac{x^2 - 4 + 3}{x^2 - 4} \\ &= \frac{x^2 - 4}{x^2 - 4} + \frac{3}{x^2 - 4} \\ &= 1 + \frac{3}{(x + 2)(x - 2)}\end{aligned}$$

Problem 6(iv):

$$\int \frac{x^2 - 1}{x^2 - 4} dx$$

Solution:

Step 1: Partial fraction

$$\begin{aligned}\frac{x^2 - 1}{x^2 - 4} &= \frac{x^2 - 4 + 3}{x^2 - 4} \\ &= \frac{x^2 - 4}{x^2 - 4} + \frac{3}{x^2 - 4} \\ &= 1 + \frac{3}{(x + 2)(x - 2)} \\ &= 1 + \frac{3}{(2 + 2)(x - 2)} + \frac{3}{(x + 2)(-2 - 2)}\end{aligned}$$

[Short Method]

Problem 6(iv):

$$\int \frac{x^2 - 1}{x^2 - 4} dx$$

Solution:

Step 1: Partial fraction

$$\begin{aligned}\frac{x^2 - 1}{x^2 - 4} &= \frac{x^2 - 4 + 3}{x^2 - 4} \\ &= \frac{x^2 - 4}{x^2 - 4} + \frac{3}{x^2 - 4} \\ &= 1 + \frac{3}{(x + 2)(x - 2)} \\ &= 1 + \frac{3}{(2 + 2)(x - 2)} + \frac{3}{(x + 2)(-2 - 2)}\end{aligned}$$

[Short Method]

$$= 1 + \frac{3}{4(x - 2)} - \frac{3}{4(x + 2)}$$

Problem 6(iv):

Step 2: Integration

$$\int \frac{x^2 - 1}{x^2 - 4} dx$$

Solution:

Step 1: Partial fraction

$$\begin{aligned}\frac{x^2 - 1}{x^2 - 4} &= \frac{x^2 - 4 + 3}{x^2 - 4} \\ &= \frac{x^2 - 4}{x^2 - 4} + \frac{3}{x^2 - 4} \\ &= 1 + \frac{3}{(x + 2)(x - 2)} \\ &= 1 + \frac{3}{(2 + 2)(x - 2)} + \frac{3}{(x + 2)(-2 - 2)}\end{aligned}$$

[Short Method]

$$= 1 + \frac{3}{4(x - 2)} - \frac{3}{4(x + 2)}$$

Problem 6(iv):

$$\int \frac{x^2 - 1}{x^2 - 4} dx$$

Solution:

Step 1: Partial fraction

$$\begin{aligned} \frac{x^2 - 1}{x^2 - 4} &= \frac{x^2 - 4 + 3}{x^2 - 4} \\ &= \frac{x^2 - 4}{x^2 - 4} + \frac{3}{x^2 - 4} \\ &= 1 + \frac{3}{(x + 2)(x - 2)} \\ &= 1 + \frac{3}{(2 + 2)(x - 2)} + \frac{3}{(x + 2)(-2 - 2)} \end{aligned}$$

[Short Method]

$$= 1 + \frac{3}{4(x - 2)} - \frac{3}{4(x + 2)}$$

Step 2: Integration

$$\text{Let, } I = \int \frac{x^2 - 1}{x^2 - 4} dx$$

Problem 6(iv):

$$\int \frac{x^2 - 1}{x^2 - 4} dx$$

Solution:

Step 1: Partial fraction

$$\begin{aligned} \frac{x^2 - 1}{x^2 - 4} &= \frac{x^2 - 4 + 3}{x^2 - 4} \\ &= \frac{x^2 - 4}{x^2 - 4} + \frac{3}{x^2 - 4} \\ &= 1 + \frac{3}{(x + 2)(x - 2)} \\ &= 1 + \frac{3}{(2 + 2)(x - 2)} + \frac{3}{(x + 2)(-2 - 2)} \end{aligned}$$

[Short Method]

$$= 1 + \frac{3}{4(x - 2)} - \frac{3}{4(x + 2)}$$

Step 2: Integration

$$\begin{aligned} \text{Let, } I &= \int \frac{x^2 - 1}{x^2 - 4} dx \\ &= \int \left(1 + \frac{3}{4(x - 2)} - \frac{3}{4(x + 2)} \right) dx \end{aligned}$$

Problem 6(iv):

$$\int \frac{x^2 - 1}{x^2 - 4} dx$$

Solution:

Step 1: Partial fraction

$$\begin{aligned} \frac{x^2 - 1}{x^2 - 4} &= \frac{x^2 - 4 + 3}{x^2 - 4} \\ &= \frac{x^2 - 4}{x^2 - 4} + \frac{3}{x^2 - 4} \\ &= 1 + \frac{3}{(x + 2)(x - 2)} \\ &= 1 + \frac{3}{(2 + 2)(x - 2)} + \frac{3}{(x + 2)(-2 - 2)} \end{aligned}$$

[Short Method]

$$= 1 + \frac{3}{4(x - 2)} - \frac{3}{4(x + 2)}$$

Step 2: Integration

$$\begin{aligned} \text{Let, } I &= \int \frac{x^2 - 1}{x^2 - 4} dx \\ &= \int \left(1 + \frac{3}{4(x - 2)} - \frac{3}{4(x + 2)} \right) dx \\ &= \int dx + \int \frac{3}{4(x - 2)} dx - \int \frac{3}{4(x + 2)} dx \end{aligned}$$

Problem 6(iv):

$$\int \frac{x^2 - 1}{x^2 - 4} dx$$

Solution:

Step 1: Partial fraction

$$\begin{aligned} \frac{x^2 - 1}{x^2 - 4} &= \frac{x^2 - 4 + 3}{x^2 - 4} \\ &= \frac{x^2 - 4}{x^2 - 4} + \frac{3}{x^2 - 4} \\ &= 1 + \frac{3}{(x + 2)(x - 2)} \\ &= 1 + \frac{3}{(2 + 2)(x - 2)} + \frac{3}{(x + 2)(-2 - 2)} \end{aligned}$$

[Short Method]

$$= 1 + \frac{3}{4(x - 2)} - \frac{3}{4(x + 2)}$$

Step 2: Integration

$$\begin{aligned} \text{Let, } I &= \int \frac{x^2 - 1}{x^2 - 4} dx \\ &= \int \left(1 + \frac{3}{4(x - 2)} - \frac{3}{4(x + 2)} \right) dx \\ &= \int dx + \int \frac{3}{4(x - 2)} dx - \int \frac{3}{4(x + 2)} dx \\ &= x + \frac{3}{4} \ln(x - 2) - \frac{3}{4} \ln(x + 2) + C \end{aligned}$$

Problem 7:

$$\int \frac{1}{e^{2x} - e^x} dx$$

Solution:

$$\begin{aligned} \text{Let, } I &= \int \frac{1}{e^{2x} - e^x} dx \\ &= \int \frac{1}{e^{2x}(1 - e^{-x})} \\ &= \int \frac{e^{-2x}}{1 - e^{-x}} dx \end{aligned}$$

$$\text{Let, } u = 1 - e^{-x}$$

$$\therefore du = e^{-x} dx$$

$$\begin{aligned} I &= \int \frac{e^{-x} e^{-x}}{1 - e^{-x}} dx \\ &= \int \frac{1 - u}{u} du \\ &= \int \left(\frac{1}{u} - 1 \right) du \end{aligned}$$

$$I = \ln u - u$$

$$= \ln(1 - e^{-x}) - (1 - e^{-x})$$

$$= \ln(1 - e^{-x}) + e^{-x} + C$$

THE END