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\documentclass[aspectratio=169,8pt]{beamer}
%\documentclass[aspectratio=169,8pt,trans]{beamer}
\usepackage{graphicx}
\usetheme{Warsaw}
\usecolortheme{seahorse}
\usefonttheme{serif}
\usepackage{amsmath}
\usepackage{xcolor}
\begin{document}
\title[Integration By Partial
Fraction]{\textcolor{red}{Lecture M100904}\!\!{
Integration By Partial Fraction}}
\author[\copyright Zarif Ahmad Razin Bhuiyan\ \ \ \
ME'16\ \ \ \ BUET]{Zarif Ahmad Razin Bhuiyan}
\institute{5\textsuperscript{th} semester, Mechanical
Engineering\!
Bangladesh University of Engineering And
Technology (BUET)}
\date{January 13, 2022}
\maketitle
\begin{frame}[t]{Exercise 10.4}
\begin{columns}[t]
\column{0.5\textwidth}
\textbf{Problem: 1(vii)}\!\!
\[\int \frac{2x+3}{x^3-x}\,dx\]
\pause
\emph{Solution:}\!\! \vspace{4pt}
Step1: Partial Fraction
\pause
\begin{align*}
\onslide<3->
{
\frac{2x+3}{x^3-x}&=\frac{2x+3}{x(x^2-1)}\!\!
&=\frac{2x+3}{x(x+1)(x-1)}\!\!
\therefore\frac{2x+3}{x^3-x}&\equiv
\frac{A}{x}+\frac{B}{x+1}+\frac{C}{x-1}\ldots(1)
}
\onslide<4->
{
\intertext{Multiplying both side by $x$ and taking limit
$x\to 0$}
\begin{split}

```

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\lim_{x\to 0}\left[\frac{2x+3}{x^3-x}\times
x\right]&=\lim_{x\to 0}\left[\frac{A}{x}\times
x\right]+\lim_{x\to 0}\left[\frac{Bx}{x+1}\right]\!\!
&\phantom{=}\lim_{x\to 0}\left[\frac{A}{x}\times
x\right]+\lim_{x\to 0}\left[\frac{Cx}{x-1}\right]
\end{split}
}
\end{align*}
\column{0.5\textwidth}
\begin{align*}
\onslide<5->{
or,&\lim_{x\to 0}\left[\frac{2x+3}{x(x+1)(x-1)}\times
x\right]&=\lim_{x\to 0}A+0+0\!\!
or,&\lim_{x\to 0}\left[\frac{2x+3}{(x+1)(x-1)}\right]&=A\!\!
or,&A=\lim_{x\to 0}\left[\frac{2x+3}{(x+1)(x-1)}\right]\!\!
or,&A=\frac{2\times 0+3}{(0+1)(0-1)}\!\!
\therefore A=-3
\intertext{Similarly, multiplying both side of (1) by
$(x+1)$ and taking limit $x\to -1$}
B=\frac{1}{2}
\intertext{and, multiplying both side of (1) by $(x-1)$
and taking limit $x\to 1$ will be found}
C=\frac{5}{2}}
\end{align*}
\end{columns}
\end{frame}

\begin{frame}[t]
\frametitle{Exercise 10.4}
\begin{columns}[t]
\column{0.5\textwidth}
\begin{align*}
\frac{2x+3}{x(x+1)(x-1)}&=\frac{-3}{x}+\frac{1}{2}\frac{1}{x+1}+\frac{5}{2}\frac{1}{x-1}
\end{align*}
\pause
Step 1: Partial Fraction

(Short Method)
\begin{align*}
\onslide<2->
{
\frac{2x+3}{x(x+1)(x-1)}&=\frac{2\times 0+3}{x(0+1)(0-1)}\!\!

```

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&+\frac{2\times (-1)+3}{(-1)(x+1)(-1-1)}\\
&+\frac{2\times 1+3}{1(1+1)(x-1)}\\
}
\onslide<3->
{
&=\frac{-3}{x}+\frac{1}{2}\frac{1}{x+1}+\frac{5}{2}\frac{1}{x-1}
}
\end{align*}
\column{0.5\textwidth}
\onslide<4->
{
Step 2: Integration\\
\begin{align*}
\text{I}&=\int \frac{2x+3}{x(x+1)(x-1)}dx\\
&=\int \left( \frac{-3}{x}+\frac{1}{2}\times \frac{1}{x+1}+\frac{5}{2}\times \frac{1}{x-1} \right) dx\\
&=-3\int \frac{1}{x}dx+\frac{1}{2}\int \frac{1}{x+1}dx+\frac{5}{2}\int \frac{1}{x-1}dx\\
&=-3\ln x+\frac{1}{2}\ln (x+1)+\frac{5}{2}\ln (x-1)+c
\end{align*}

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\end{columns}
\end{frame}
\begin{frame}[t]
\frametitle{Exercise 10.4}
\begin{columns}[t]
\column{0.5\textwidth}
\textbf{Problem 2(v):}
[
\int \frac{2x+3}{x^3+x^2-2x}dx
]
\onslide<1->
{
\emph{Solution:}\\
\vspace{4pt}
Step 1: Partial Fraction
\vspace{0pt}
\begin{align*}
\frac{2x+3}{x^3+x^2-2x}&=\frac{2x+3}{x(x^2+x-2)}\\
&=\frac{2x+3}{x(x(x+2)-1(x+2))}
\end{align*}
\onslide<2->
{
&=\frac{2x+3}{x(x(x+2)-1(x+2))}
}
\onslide<3->
{
&=\frac{2x+3}{x(x(x+2)-1(x+2))}
}
\onslide<4->
{
&=\frac{2x+3}{x(x(x+2)-1(x+2))}
}
\onslide<5->
{
&\equiv \frac{2\times 0+3}{x(0+2)(0-1)}+\frac{2\times (-2)+3}{(-2)(x+2)(-2-1)}+\frac{2\times 1+3}{1(1+2)(x-1)}
\end{align*}
\column{0.5\textwidth}
\begin{align*}
&\frac{2x+3}{x^3+x^3-

```

```

\frac{3}{2}\{x\}+\frac{\frac{1}{6}\{x+2\}+\frac{\frac{5}{3}\{3\}\{x-1\}}
}
\end{align*}
\onslide<7->
{
Step 2: Integration
}
\begin{align*}
\onslide<7->
{
\text{Let, } I&=\int
\frac{2x+3}{x^3+x^2-2x}\,dx\\
}
\onslide<8->
{
&=\int \left(\frac{-
\frac{3}{2}\{x\}+\frac{\frac{1}{6}\{x+2\}+\frac{\frac{5}{3}\{3\}\{x-1\}}{x-1}\right)\,dx\\
}
\onslide<9->
{
&=\int \frac{-
\frac{3}{2}\{x\}\,dx+\int
\frac{\frac{1}{6}\{x+2\}\,dx+\int \frac{\frac{5}{3}\{3\}\{x-1\}\,dx\\
}
\onslide<10->
{
&=\frac{-3}{2}\int
\frac{1}{x}\,dx+\frac{1}{6}\int
\frac{1}{x+2}\,dx+\frac{5}{3}\int \frac{1}{x-1}\,dx\\
}
\onslide<11->
{
&=\frac{-3}{2}\ln
x+\frac{1}{6}\ln (x+2)+\frac{5}{3}\ln (x-1)+c
}
\end{align*}
\end{columns}
\end{frame}
\begin{frame}[t]
\frametitle{Exercise 10.4}
\begin{columns}[t]
\column{0.5\textwidth}
\textbf{Problem 3(iii):}
\[

```

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\int \frac{x}{(x-
1)^2(x+2)}\,dx

\]
\emph{Solution:}\\
\onslide<2->
{
Step 1: Partial Fraction
\vspace{0pt}
}
\begin{align*}
\onslide<3->
{
\frac{x}{(x-
1)^2(x+2)}&\equiv \frac{A}{x-1}+\frac{B}{(x-
1)^2}+\frac{C}{x+2}+\ldots(1)
}
\onslide<4->
{
\intertext{Multiplying both
side of (1) by $(x-1)^2$ and taking limit $x\to 1$,}
}
\onslide<5->
{
\lim_{x\to 1}\left[
\frac{x(x-1)^2}{(x-1)^2(x+2)}\right]&=\lim_{x\to 1}
\left[\frac{A(x-1)^2}{(x-1)}\right]\\
&+\lim_{x\to
1}\left[\frac{B(x-1)^2}{(x-1)^2}\right]\\
&+\lim_{x\to 1}
\left[\frac{C(x-1)^2}{x+2}\right]
}
\end{align*}
\column{0.5\textwidth}
\begin{align*}
\onslide<6->
{
&\Rightarrow \lim_{x\to
1} \left[\frac{x}{x+2}\right]&=\lim_{x\to 1} \left[
A(x-1)\right]+\lim_{x\to 1}[B]\\
&\quad\quad\quad&+\lim_{x\to
1}\left[\frac{C(x-1)^2}{x+2}\right]
}
\end{align*}
\onslide<7->
{
&\Rightarrow
\frac{1}{1+2}=B\\
}
\onslide<8->
{
&\text{therefore } B=\frac{1}{3}
}

```

```

    }
\end{align*}
\onslide<9->
{
Short Method: (Applicable for B and
C only)
}
\begin{align*}
\onslide<10->
{
      B&=\frac{1}{(x-
1)^2(1+2)}=\frac{1}{3}\backslash\backslash
    }
\onslide<11->
{
      C&= \frac{-2}{(-2-
1)^2(x+2)}=\frac{-2}{9}
    }
\end{align*}
\onslide<12->
{
Putting  $x=0$  on the both side of
(1),
}
\begin{align*}
\onslide<13->
{
      0=A+B\frac{C}{2}
\quad \text{therefore } A=-
\frac{2}{9}
    }
\end{align*}
}
\end{columns}
\end{frame}

\begin{frame}[t]
\frametitle{Exercise 10.4}
\begin{columns}[t]
\column{0.5\textwidth}
\onslide<1->
{
      Step 2: Integration
    }
\begin{align*}
\onslide<2->
{

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\frac{x}{(x-1)^2(x+2)}\backslash, dx\backslash\backslash
    }
\onslide<3->
{
      &=\int \left(\frac{-
\frac{2}{9}}{x-1}+\frac{\frac{1}{3}}{(x-1)^2}+\frac{-
\frac{2}{9}}{x+2}\right)\backslash, dx\backslash\backslash
    }
\onslide<4->
{
      &=-\frac{2}{9}\int
\frac{1}{x-1}\backslash, dx+\frac{1}{3}\int \frac{1}{(x-1)^2}\backslash, dx-
\frac{2}{9}\int \frac{1}{x+2}\backslash, dx\backslash\backslash
    }
\onslide<5->
{
      &=-\frac{2}{9}\ln (x-
1)+\frac{1}{3}\frac{(x-1)^{-2+1}}{-2+1}-\frac{2}{9}\ln
(x+2)+C\backslash\backslash
    }
\onslide<6->
{
      &=-\frac{2}{9}\ln (x-1)-
\frac{1}{3}\frac{1}{x-1}-\frac{2}{9}\ln (x+2)+C
    }
\end{align*}
\end{columns}
\end{frame}

\begin{frame}[t]
\frametitle{Exercise 10.4}
\begin{columns}[t]
\column{0.5\textwidth}
\textbf{Problem 4(iii) :}
\left[
      \int \frac{x+2}{(1-
x)(x^2+4)}\backslash, dx
    \right]
\pause
\onslide<2->
{
\emph{Solution:}\backslash\backslash
\vspace{4pt}
Step1: Partial Fraction
\vspace{0pt}
}
\begin{align*}
\onslide<3->

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{
    \frac{x+2}{(1-x)(x^2+4)} \equiv \frac{A}{1-x} + \frac{Bx+C}{x^2+4} \dots (1) \\
}
\onslide<4->
{
    \intertext{multiplying both side of (1) by $(1-x)$ and taking limit $x \to 1$,}
}
\onslide<5->
{
    \lim_{x \to 1} \left[ \frac{(x+2)(1-x)}{(1-x)(x^2+4)} \right] \equiv \lim_{x \to 1} \left[ \frac{A(1-x)}{(1-x)} \right] \\
    + \lim_{x \to 1} \left[ \frac{(Bx+C)(1-x)}{x^2+4} \right] \\
}
\onslide<6->
{
    \Rightarrow \lim_{x \to 1} \left[ \frac{x+2}{x^2+4} \right] \equiv \lim_{x \to 1} [A] + \lim_{x \to 1} \left[ \frac{(Bx+C)(1-x)}{x^2+4} \right]
}
\end{align*}
\column{0.5\textwidth}
\begin{align*}
\onslide<7->
{
    \Rightarrow \frac{1+2}{1^2+4} \equiv A+0 \\
    \therefore A = \frac{3}{5}
}
\end{align*}
\onslide<8->
{
    Now, multiplying both side of (1) by $x$ and taking limit $x \to \infty$,
}
\begin{align*}
\onslide<9->
{
    & \lim_{x \to \infty} \left[ \frac{(x+2)x}{(1-x)(x^2+4)} \right] \equiv \lim_{x \to \infty} \left[ \frac{Ax}{1-x} \right] \\
    & + \lim_{x \to \infty} \left[ \frac{(Bx^2+cx)}{x^2+4} \right]
}
\end{align*}

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}
\onslide<10->
{
    \Rightarrow 0 \equiv -
}
A+B \\
A=B = \frac{3}{5}
}
\end{align*}
\onslide<11->
{
    \emph{Note:} You can use short method (See Problem 1(vii)) for finding 'A'
}
\end{columns}
\end{frame}
\begin{frame}[t]
\frametitle{Exercise 10.4}
\begin{columns}[t]
\column{0.5\textwidth}
    Again, put $x=0$ in (1),
    \begin{align*}
        \onslide<1->
        {
            \frac{0+2}{0^2+4} \equiv \frac{A}{1-0} + \frac{B \times 0 + C}{0^2+4} \\
        }
        \onslide<2->
        {
            \Rightarrow
            \frac{1}{2} \equiv A + \frac{C}{4}
        }
        \onslide<3->
        {
            \Rightarrow
            \frac{1}{2} \equiv \frac{3}{5} + \frac{C}{4}
        }
        \onslide<4->
        {
            \therefore
            C = \frac{-2}{5}
        }
    }
\end{align*}
\onslide<5->
{
    Step 2: Integration
}

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```

}
\onslide<6->
{
\begin{align*}
\text{Let,}
I&=\int \frac{x+2}{(1-x)(x^2+4)}dx\\
&
\end{align*}
\onslide<7->
{
&=\int
\left( \frac{\frac{3}{5}}{1-x}+\frac{\frac{3}{5}x-
\frac{2}{5}}{x^2+4} \right)dx\\
&
\end{align*}
\onslide<8->
{
&=\int
\frac{\frac{3}{5}}{1-x}dx+\int \frac{\frac{3}{5}x-
\frac{2}{5}}{x^2+4}dx\\
&
\end{align*}
\column{0.5\textwidth}
\begin{align*}
&
\end{align*}
\onslide<9->
{
&=\frac{3}{5}\int \frac{1}{1-x}dx+\int
\frac{\frac{3}{5}x}{x^2+4}dx+\int \frac{-
\frac{2}{5}}{x^2+4}dx\\
&
\end{align*}
\onslide<10->
{
&=
\frac{3}{5}\ln (1-x) +\frac{3}{5}\int
\frac{\frac{1}{2}\times 2x}{x^2+4}dx-\frac{2}{5}\int
\frac{1}{x^2+2^2}dx\\
&
\end{align*}
\onslide<10->
{
&=
\frac{3}{5}\ln(1-x)+\frac{3}{5}\times \frac{1}{2} \times
\ln (x^2+4)-\frac{2}{5}\times \frac{1}{2}\arctan
\frac{x}{2}\\
&
\end{align*}
\onslide<11>
{
&=
\frac{3}{5}\ln (1-x) +\frac{3}{10}\ln(x^2+4)-
\frac{1}{5}\arctan \frac{x}{2}+c
}

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\end{align*}
\end{columns}
\end{frame}

\begin{frame}[t]
\frametitle{Exercise 10.4}
\begin{columns}[t]
\column{0.5\textwidth}
\onslide<1->
{
\textbf{Problem 5(i):}
\int
\frac{2x+1}{(2x+3)^2}dx
\]
\emph{Solution:}
}
\onslide<2->
{
Step 1: Partial Fraction
\begin{align*}
\frac{2x+1}{(2x+3)^2} &\equiv
\frac{A}{(2x+3)} + \frac{B}{(2x+3)^2} \quad (1)
\end{align*}
\onslide<3->
{
Short Method:
}
\onslide<4->
{
\begin{align*}
B &= \frac{-3}{2} + 1 = -\frac{1}{2}
\end{align*}
}
\onslide<5->
{
Putting  $x=0$  in (1),
\begin{align*}
\frac{-3}{2} + 1 &= -\frac{1}{2}
\end{align*}
}
\onslide<6->
{

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\frac{2\times
0+1}{(2\times 0+3)^2}&=\frac{A}{2\times 0+3}+\frac{-
2}{(2\times 0+3)^2} \\
}
\onslide<7->
{
\frac{1}{9}&=\frac{A}{3}-\frac{2}{9}\\
}
\onslide<8->
{
\therefore A&=1
}
\end{align*}
\column{0.5\textwidth}
\onslide<9->
{
Step 2: Integration
}
\begin{align*}
\onslide<10->
{
\text{Let,}
I&=\int \frac{2x+1}{(2x+3)^2}\,dx\\
}
\onslide<11->
{
&=\int
\left(\frac{1}{(2x+3)}-\frac{2}{(2x+3)^2}\right)\,dx\\
}
\onslide<12->
{
&=\int
\frac{1}{2x+3}\,dx-\int \frac{2}{(2x+3)^2}\,dx\\
}
\onslide<13->
{
&=\frac{1}{2}\ln
(2x+3)-2\times \frac{1}{2}\times \frac{(2x+3)^{-
2+1}}{-2+1}\\
}
\onslide<14->
{
&=\frac{1}{2}\ln
(2x+3)+\frac{1}{2x+3}+C
}

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\end{align*}
\end{columns}
\end{frame}
\begin{frame}[t]
\frametitle{Exercise 10.4}
\begin{columns}[t]
\column{0.5\textwidth}
\textbf{Problem 6(iv):}
\int \frac{x^2-
1}{x^2-4}\,dx
\]
\emph{Solution:}\\
\vspace{4pt}
\onslide<2->
{
Step 1: Partial fraction
}
\onslide<3->
{
\begin{align*}
&\frac{x^2-1}{x^2-
4}&= \frac{x^2-4+3}{x^2-4}\\
&\onslide<4->
{
&=\frac{x^2-
4}{x^2-4}+\frac{3}{x^2-4}\\
&\onslide<5->
{
&=1+\frac{3}{(x+2)(x-2)}\\
&\onslide<6->
{
&=1+\frac{3}{(2+2)(x-2)}+\frac{3}{(x+2)(-2-2)} \\
&\phantom{=1+}\frac{3}{(2+2)(x-2)}\}\text{[Short
Method]}\\
&\onslide<7->
{

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                &=1+\frac{3}{4}(x-
2)-\frac{3}{4}(x+2)
            }
            \end{align*}
            \column{0.5\textwidth}
            \onslide<8->
            {
                Step 2: Integration
            }

            \begin{align*}
            \onslide<9->
            {
                \text{Let,} I&=
\int \frac{x^2-1}{x^2-4} \, dx \\
            }
            \onslide<10->
            {
                &=\int
\left(1+\frac{3}{4}(x-2)-\frac{3}{4}(x+2)\right) \, dx \\
            }
            \onslide<11->
            {
                &=\int \, dx +\int
\frac{3}{4}(x-2) \, dx -\int \frac{3}{4}(x+2) \, dx \\
            }
            \onslide<12>
            {
                &=x+\frac{3}{4}\ln
(x-2) -\frac{3}{4}\ln (x+2) +C
            }
            \end{align*}
        \end{columns}
    \end{frame}

    \begin{frame}[t]
        \frametitle{Exercis 10.4}
        \begin{columns}[t]
            \column{0.5\textwidth}
            \textbf{Problem 7:}
            \[
                \int
\frac{1}{e^{2x}-e^x} \, dx
            \]
            \emph{Solution:}
        \end{columns}
    \end{frame}

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        \text{Let,}
        I&=\int \frac{1}{e^{2x}-e^x} \, dx \\
        \frac{1}{e^{2x}(1-e^{-x})} \\
        2x)\{1-e^{-x}\} \, dx \\
        e^{-x} \\
        du&=e^{-x}dx \\
        x)e^{-x}\{1-e^{-x}\} \, dx \\
        u\}u\} \, du \\
        &=\int
\left(\frac{1}{u}-1\right) \, du
        \end{align*}
        \column{0.5\textwidth}
        \begin{align*}
            I&=\ln u - u \\
            &=\ln(1-e^{-x}) -
&=\ln(1-e^{-
x))+e^{-x}+C
        \end{align*}
    \end{columns}
\end{frame}
\setbeamertemplate{footline}[default]
\begin{frame}
    \begin{figure}
        \centering
        {\Huge THE END}
        %\centering
        %\includegraphics[scale=0.45]{Logo}
    \end{figure}
\end{frame}
\end{document}

```